

MODEL PAPER
II PUC MATHEMATICS(35)

Time : 3 hours

Max. Marks : 100

PART - A

Answer ALL the questions.

10X1=10

1. Find the number of solutions of $5x \equiv 3 \pmod{15}$
2. If $\vec{a} = 2i+3j+6k$ and $\vec{b} = i-2j+2k$, find the projection of \vec{a} on \vec{b}
3. If $\begin{pmatrix} 0 & 2x-6 \\ 3x-4 & 0 \end{pmatrix}$ is a skew symmetric matrix, find x.
4. If $a*b = a+b-5 \quad \forall a, b \in I$, find the identity.
5. Find the position of the point (2, -3) with respect to the circle $x^2+y^2-2x-6y-11=0$
6. Find the axis of the parabola $y^2+x-2y+4=0$
7. Evaluate $\cos[2\sin^{-1}(1/3)]$
8. Prove that $\left(\cos \frac{3\pi}{7} + i \sin \frac{3\pi}{7} \right)^{\frac{7}{2}} = -i$
9. If $f(x) = \tan^{-1}(\sqrt{x^2+1})$ then find $f'(1)$
10. Evaluate $\int_{-1}^1 x^2 \sin(x^3) dx$

PART -B

Answer any TEN questions

10X2=20

11. If p is a prime number and p|ab then prove that p|a or p|b
12. Find the value of λ so that the vectors $3i-j-\lambda k$, $i-j+k$ and $2i-4j+3k$ are coplanar
13. If $A(\text{adj}A) = |A|I = (\text{adj}A)A$, Prove that $(\text{adj}A)^{-1} = \frac{A}{|A|}$
14. In a group G if $(ab)^{-1} = a^{-1}b^{-1}$ Show that G is an abelian.
15. Show that radical axis of two circles is perpendicular to line joining the centers of circles
16. Any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ makes intercepts h and k on the co-ordinate axes. Show that $\frac{a^2}{h^2} + \frac{b^2}{k^2} = 1$
17. If $\sin^{-1} \frac{5}{x} + \sin^{-1} \frac{12}{x} = \frac{\pi}{2}$ find x.
18. Find the modulus and amplitude of complex number $2-2i - \sqrt{3}$
19. If $e^x + e^y = e^{x+y}$, Prove that $\frac{dy}{dx} = -e^{y-x}$
20. If $y^2 = 8kx$ and $xy = 2p$ intersect orthogonally, Prove that $p^2 = 128k^4$
21. Integrate $\frac{1}{\sqrt{6-x-x^2}}$ w.r.t x
22. Form the differential equation of the circle passing through the origin and having their centre on the line $y=2x$

PART-C

Answer any THREE questions

3X5=15

23. Find the GCD of 495 and 675 and find k and l such that $495k+675l$ and show that k and l are not unique. 5
24. a) Prove that $\sin(A+B) = \sin A \cos B + \cos A \sin B$ by vector method 3
b) Show that $(4\vec{a} + 3\vec{b}) \times (2\vec{a} + 3\vec{b}) = 6(\vec{a} \times \vec{b})$ 2
25. Prove that the set of all +ve rational numbers forms an abelian group w.r.t multiplication * defined by $a*b = ab/6$ and hence solve $x*3^{-1} = 2$. 5
26. Solve by matrix method:
 $3x+2y-z=4$, $x-y+4z=11$, $2x+y-z=1$. 5

Answer any TWO questions

2X5=10

27. Find the equation of the circle with center on $2x+3y=7$ and cutting orthogonally circles $x^2 + y^2 - 10x - 4y + 21 = 0$ and $x^2 + y^2 - 4x - 6y + 11 = 0$ 5
28. Find the centre, eccentricity and ends of the latus rectum of the hyperbola $9x^2 - 16y^2 - 18x - 64y + 89 = 0$ 3
 b) Find k if $y=3x+k$ touches the ellipse $4x^2 + 9y^2 = 36$ 2
29. a) Find $\sin(\cos^{-1} 1/3 - \sin^{-1} 2/3)$ 3
 b) Find the general solution of $\cos 2x + \cos 3x = 0$ 2

Answer any THREE questions

3X5=15

30. a) Differentiate $\sin x$ by first principles 3
 b) Differentiate $\log_a x$ with reference to $\log_x a$
31. a) If $y = \cos m\theta$, $x = \cos \theta$, Prove that $(1-x^2)y_2 - xy_1 + m^2y = 0$ 3
 b) Integrate $\frac{x^2}{4x^6 + 1}$ w.r.t x 2
32. a) Integrate $\frac{3\cos x + \sin x}{4\cos x + 3\sin x}$ w.r.t x 3
 b) Find the angle between the curves $x^2 + y^2 + 3x - 8 = 0$ and $x^2 + y^2 = 5$ at (1,2) 2
33. a) Differentiate $\tan^{-1}\left(\frac{1+3x}{1-3x}\right)$ w.r.t $\sin^{-1}\left(\frac{6x}{1+9x^2}\right)$ 3
 b) Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\cos^3 x + \sin^3 x}$ 2
34. Find the area enclosed between the parabola $x^2 = 4y$ and the line $x = 4y - 2$ 5

PART-D

Answer any TWO questions

35. a) Prove that $[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2$ 3
 b) If $\vec{a} = i + j - k$, $\vec{b} = i - 3j + k$, $\vec{c} = 3i - 4j + 2k$ find $(\vec{a} \times \vec{b}) \times \vec{c}$ 3
- b) Prove that $\begin{vmatrix} x & x^2 & y+z \\ y & y^2 & z+x \\ z & z^2 & x+y \end{vmatrix} = (x-y)(y-z)(z-x)(x+y+z)$ 4
36. a) Derive the equation of the hyperbola in the standard form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, Write equation to the locus of the point of intersection of perpendicular tangents to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and write the name of the equation.. 6
 b) Find the general solution of trigonometric equation $\sqrt{3} \tan x = \sqrt{2} \sec x - 1$ 4
37. a) A man 6 feet in height moves away at a uniform rate of 4m.p.h. From a source of light which is 20 feet above the ground. Find the rate at which the shadow lengthens and the rate at which the tip of his shadow is moving. 6
 b) Solve the equation $(y^2 + y)dx + (x^2 + x)dy = 0$ given that $y=2$ and $x=1$ 4
38. a) If $\cos \alpha = \cos \beta = \cos \gamma = 0$, Prove that
 i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3 \cos(\alpha + \beta + \gamma)$
 $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3 \sin(\alpha + \beta + \gamma)$
 ii) $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = \sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = \frac{3}{2}$ 6
- b) Show that $\int_0^{\frac{\pi}{2}} \log \sin x \, dx = -\frac{\pi}{2} \log 2$ 4

PART -E

Answer any ONE question

39. a) Find all the fourth roots of $1 + i\sqrt{3}$ 4
 b) Find the length of the common chord of intersecting circles $x^2 + y^2 - 4x - 5 = 0$ and $x^2 + y^2 - 2x + 8y + 9 = 0$ 4
 c) Find the remainder when 5^{20} is divided by 7 2
40. a) Show that maximum rectangle that can be inscribed in a circle is a square 4
 b) Evaluate $\int \operatorname{cosec}^3 x \, dx$ 4
 c) Find the order and degree of Differential equation $\frac{d^2 y}{dx^2} = \left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{\frac{1}{2}}$ 2

